

Towards Tractable Stochastic Unit Commitment for Preventive Operation during Hurricanes¹

Farshad Mohammadi and Mostafa Sahraei-Ardakani
Department of Electrical and Computer Engineering
University of Utah
Salt Lake City, USA

Abstract—Hurricanes cause power outages in the coastal areas of the United States every year. Previous studies show that the integration of weather forecast data within power system operation can substantially reduce such outages. However, preventive power system operation, modeled as a stochastic unit commitment, is extremely computationally burdensome, and, thus not applicable to real systems. The present paper aims to address this challenge by developing a computationally-efficient stochastic unit commitment model. The proposed algorithm proactively schedules generation units during hurricanes, taking the hurricane-induced damages into consideration. The proposed model was able to solve a large-scale 2000-bus Texas system in 7 hours, achieving an acceptable level of computational tractability. In the simulated hurricane, the proposed model was also able to avoid 80% of the power outages by only adding 5% to the dispatch cost. The results confirm the viability of stochastic unit commitment as a preventive operation tool to reduce the power outages during hurricanes.

Index Terms—Stochastic unit commitment, hurricane, power system reliability, transmission outage, preventive operation, large-scale systems, load shedding, power outage.

I. INTRODUCTION

Hurricane is among nature's most destructive and powerful disasters causing substantial discomfort every year [1][2]. Seasonal hurricanes are common to hit the east coast of the U.S. just below twice on average every year and cause billions of dollars in damages to infrastructure, including power system. After a hurricane hits, utility crews are dispatched towards the affected region to repair the damages and minimize the power outages. Timely restoration is necessary and vital for the people, who live in the affected area. In 2005, hurricane Katrina destroyed 181 power lines, 3,478 transformers and over 263 substations [3]. In 2012, hurricane Isaac was responsible for the destruction of 95 transmission lines, and over 144 substations led to over 1 million homes to lose their power for days while over 12,000 workers were trying to restore the power [4]. Improving the resilience of the power system during hurricanes can bring about the substantial level of relief to the society and alleviate its disastrous consequences. Given the substantial level of damage to power system components, this paper aims to study if the final impacts, regarding a power outage, can be reduced using improved operation models.

The ability to evaluate the power network before the hurricane landfall can help improve the operator preparedness in advance of emergency [5]. Different statistical and

analytical methods are being used to evaluate the performance and reliability of the power system in confronting with severe weather conditions. Statistical models use historical outage statistics and reliability data to estimate power outages in the future when physical data and prediction of weather condition are available [6]. Some researchers have focused on the estimation of the failure rate of different components [7] and attempted to estimate system reliability by using such information [8][9]. Authors of [10] claim that it is possible to reduce the risks and failure rate by having dynamic maintenance and maintain the old components. Same authors have also proposed a cost-effective framework, which facilitates the repair and restoration of the power system for the IEEE 118-bus system [11].

Probabilistic nature of the failure of components in severe weather condition makes it natural to use stochastic programming. The major problem with stochastic programming and the vast size of power network affected by the hurricane is the computation burden and for most cases the required hardware capabilities such as available memory to store the problem and solve it. From the very first implementation of stochastic programming in power network in 1996, researchers have relied on model simplifications and scenario selection to reduce the calculation requirements, even for educational size systems such [12], [13]. There also exist studies, where enhanced the formulations are used, which take into account all sources of uncertainty in the network operation [14]–[16]. Recent research proposes an efficient approach to use stochastic programming in large systems [17]. However, this is mostly limited to the uncertainty in renewable power generation and special assumptions related to their generation. Despite the enhancements over the past two decades, problem size and the computation burden remain to be the main challenge of implementation for real-world large-scale systems. In the specific case of preventive power system operation during hurricanes, our previous work [18]–[20] faced similar challenges. We were able to show that stochastic optimization can effectively reduce power outages; however, the results were obtained for small-scale systems and the solution time was extremely long. The present paper aims to address these challenges.

To do so, this paper develops a method to utilize hurricane data (through the weather forecast) within the power system scheduling (unit commitment) in order to reduce the power outage at the minimum cost during a hurricane. The processing and memory requirements stay within the acceptable range for

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a large-scale real-world network in the presence of hurricane effects in a vast region.

Three steps should be taken to generate meaningful results: hurricane forecast in the area of study, probabilistic estimation of the component outages, and finally solving the stochastic unit commitment problem. This paper exclusively focuses on the third step. The objective is to determine the day-ahead dispatch of generation units to achieve minimum cost and minimum power outage.

The rest of this paper is organized as follows: in section II, hurricanes and their effect on the power system is discussed. Section III describes and formulates the problem. In section IV, the case study is presented. Finally, in section V, the results and conclusion are discussed.

II. HURRICANES AND POWER NETWORKS

June 1 to November 30 is the official hurricane season for Atlantic Basin. Millions of people are left without power for days and in some cases weeks during this season. Hurricane is the most severe cyclone in term of sustained wind speed with the minimum wind speed of 74 mph [21].

Even a weak hurricane can cause damage to the transmission and distribution networks and result in a power outage. Hurricanes and storms can damage or uproot transmission and distribution poles, especially when soil becomes saturated with water, which is shared with heavy hurricane precipitation. Flying objects carried by the wind can also hit poles and cables directly and damage transmission lines. While the overhead transmission and distribution networks are vulnerable to hurricanes and even storms, it is unlikely that hurricanes damage power plants directly [22]. This paper focuses on day-ahead generation scheduling during hurricanes, while taking the likelihood of hurricane-induced line outages into account. Note that it is possible that a generation unit, gets partially disconnected from the network due to damaged transmission lines. Line outage in this study is modeled as a time-dependent probability for each line, calculated based on hurricane forecast and simulation of power poles in ANSYS software.

III. PROBLEM DESCRIPTION

Stochastic unit commitment is a computationally demanding problem even for the small systems. The number of uncertainties can affect this complexity. In this paper, the objective is to solve the unit commitment problem and determine the best generation schedule in a way that the generation cost and lost load (load shedding) are minimal. The stochastic unit commitment problem should be solved in the presence of uncertainties caused by the hurricane. Moreover, scenarios regarding the uncertainties should be identified as a part of input information.

Hurricanes sweep the area of study and impact different power system components. As mentioned, the generation units are not a concern in this study, and the main source of uncertainty is the status of transmission lines. As the hurricane passes through the area, it hits different lines at different times with the different level of strength. As a result, the failure model, used in this paper, identifies probabilistic line statuses

during each period. For each line in the path of the hurricane, there is a vector that illustrates the possibility of line outage over time. The final table includes all the affected lines with the hourly possibility of outage for each line.

In order to achieve the objective of this paper, a preventive optimization algorithm is developed. The algorithm is based on a DC power flow unit commitment (UC) formulation, considering contingencies caused by transmission line outages. Overgeneration and load shedding are allowed and penalized with a high cost in the objective function. Using this model, a preventive operation plan can be obtained for the day-ahead market to reduce penalties, caused by load shedding or over generation when extreme weather events like hurricanes occur.

The general formulation of the problem is shown in (1) to (6). In these equations, indices: s , t , g , b and l stand for scenario number, time, generation unit number, bus number and line number, variables: c_{ls} , c_o , π , u , p , ls , d , o , and lf represent load shedding cost, over-generation cost, probability of scenario, binary generation commitment status, power generation, load shedding, demand, over-generation value, and line flow, and χ is a corresponding equation for generation cost, respectively. The objective function is expressed by (1), which minimizes the dispatch cost of the system considering generation dispatch, over-generation and load shedding. Note that χ covers all the costs related to generation including marginal fuel cost, no load cost, start-up and shut-down cost. Load balance, considering the load shedding and over-generation is expressed by (2), and generation limits by (3). (4) forces the optimization to have the same commitment for every generation unit over all the scenarios, and (5) expresses the thermal line flow constraints. Equation (6) represents all other common unit commitment constraints including generation ramp-up/down limits, minimum up and down time for generators, and line flow constraints. Since contingencies are modelled explicitly, reserves are not modeled in this formulation.

$$\min \sum_s \pi^s [\sum_{t,g} [u_{t,g}^s \chi_g(p_{t,g}^s)] + \sum_b c_{ls}^s \cdot ls_{t,b}^s + \sum_b c_o^s \cdot o_{t,b}^s] \quad (1)$$

Subject to,

$$\sum_g p_{t,g}^s + \sum_b ls_{t,b}^s = \sum_b d_{t,b}^s + \sum_g o_{t,g}^s \quad \forall t, s \quad (2)$$

$$u_{t,g}^s p_{t,g}^s \leq p_{t,g}^s \leq u_{t,g}^s \bar{p}_g^s \quad \forall t, s, g \quad (3)$$

$$u_{t,g}^s = u_{t,g} \quad \forall t, s, g \quad (4)$$

$$lf_l^s \leq lf_{t,l}^s \leq \bar{lf}_l^s \quad \forall s, l \quad (5)$$

$$(u, p) \in \mathcal{E} \quad (6)$$

As the topology of network changes over the time (due to the line outage because of the hurricane), if large-scale network data applied to (1) to (6) and line flow calculations be done by using conventional B0 formulation, the number of variables and constraints even for one single scenario would be enormous and the problem would be almost impossible to solve using most of the state of the art workstations. On the other hand, in the creation of scenarios, each line can be online or offline at different times. As an example, considering a total

number of 100 lines out of thousands affected by the hurricane, the total number of possible scenarios would be $2^{100 \times 24}$. It is impossible to model all these scenarios, as it would result in extreme computational burden, beyond the capabilities of our computers.

Using sensitivity factors and flow-canceling transactions, as described in [23], can reduce the computational complexity and number of variables significantly, especially in the case with line outages. Flow-canceling transactions are injection pairs at the two end of a transmission line that can emulate a line outage, without affecting the network topology and shift factors. Including flow canceling transactions the rest of the network would see the line as open, while the shift factors remain unchanged. The flow-canceling transaction for line k , v_k , can be calculated by using (7). In (7), $\varphi_k^{k_{from/to} \text{ bus}}$ represents the *injection shift factor* related to line k (refer to [24] for further information about how to calculate the injection shift factors for the network).

In common application, because of dependency of v_k to the line flow before outage, this method works for a single line outage and the simple superposition does not apply. As the number of line outages is more than one for the purpose of this paper, it is necessary to adjust the flow-canceling transactions to be accurate with multiple line outages. To do so, the original flow-canceling equation, as shown in (8), is applied for each scenario and every hour as a constraint to the main optimization problem. In (8), “ o ” is a set of all offline lines. As an example, if “ o ” includes three outages, a set of linear equations with three equations and three unknowns would be appeared in constraints. Including these equations in the optimization problem as constraints, will solve the equations simultaneously. This method is successfully used in the existing literature for optimal transmission switching [23]. If the only line in set “ o ”, is k , then (8) can be simplified to (7).

$$lf_k - v_k \cdot [1 - [\varphi_k^{k_{from} \text{ bus}} - \varphi_k^{k_{to} \text{ bus}}]] = 0 \quad (7)$$

$$lf_k - v_k + \sum_o [\varphi_o^{k_{from} \text{ bus}} - \varphi_o^{k_{to} \text{ bus}}] \cdot v_o = 0 \quad (8)$$

Even by using sensitivity factor method (such as Line Outage Distribution Factor, LODF) and flow canceling transactions, the problem size for the real-world network is huge. According to benchmarks we have studied for different sizes of networks, a big portion of calculation power and memory usage is dedicated over the variables and constraints related to line flows. In (1) to (6) and (8), every line will be monitored for every hour to avoid the thermal capacity violation. Hence, an iterative linear optimization algorithm is designed to help prevent any unnecessary constraint and variable. Fig. 1 illustrates the flowchart of calculation.

In Fig. 1 flowchart, “A” represents the data reading from files; “B” includes initial basic calculations, data conversions, and determination of appropriate dimension for variables; “C” checks the input data to find any error or incompatible sizing; “D” calculates the shift factor arrays for the original topology of the network without any outage, and LODF matrix

corresponding to line outage set for different scenarios; “E” defines decision variables and their size alongside with initial parameters including acceptable tolerance for IBM CPLEX Studio [25]; “E” defines the objective function needed to be minimized; “G” adds generation, commitments, load balance, and scenario constraints; “H” adds constraints regarding the monitored line set and outage set for each scenario and hour; “I” solves the minimization and generate temporarily/final results; “J” calculates line flow for every line, hour and scenario, while “K” decides which line to be monitored for the next iteration; “L” removes lines from monitored set if they are out for some hours and “M” helps the CPLEX to reduce the number of constraints if they are not needed any longer.

IV. CASE STUDY AND BENCHMARK

The network which used in this paper to evaluate the designed algorithm is ACTIVSg2000: a 2000-bus synthetic grid on the footprint of Texas [26], [27]. There are 2000 buses, 3,206 branches, and 544 generation units.

A hypothetical hurricane is assumed to pass through the region within 24 hours. The affected area is a circle with the radius equal to the distance hurricane travel in four hours. The path of the hurricane, as well as the location of buses and the affected area, is illustrated in Fig. 2. A total number of critical affected lines is 47. For each of these 47 lines, there is a possibility of being damaged by a hurricane every hour. Fig. 3, illustrates the example of outage possibility for a number of selected lines. In this figure, line 702 is the first line hit by hurricane and line 2,959 is the last.

To analyze the performance of the proposed algorithm, the unit commitment problem is solved by using different standard methods. The unit commitment is solved for the original network without hurricane and line outage. Results are presented in TABLE 1. These results show represent business as usual (BAU). All the simulations have been performed with the same workstation (Intel® Core™ i7-7700 CPU @ 3.60 GHz, 16 GB of DDR4 RAM, LITEON CV3-8D512 SSD) and software (ECLIPS IDE 4.9 and IBM CPLEX 12.8). As it is obvious from TABLE 1, the proposed algorithm is as accurate as other methods and about 70 times faster than B0 method.

Solving the stochastic unit commitment in the case with a hurricane and line outage uncertainty is an objective of this paper to prove the performance and application of the proposed algorithm. Scenario reduction is performed by having different thresholds in possibilities for different scenarios. For the first scenario, the threshold is 1% which means any chance of line outage more than 1% would be considered as a certain outage in this scenario (worst-case scenario). For the 10th scenario, best-case scenario, the threshold is considered 100% which means only those lines which we are sure about their failure would be considered offline, and the rest of lines are assumed in-service. The thresholds for other scenarios are as 50%, 60%, 70%, 75%, 80%, 85%, 90%, and 95%, respectively.

Neither B0 nor standard shift factor was able to solve the stochastic problem, due to memory issues. We also tried to run them on a more powerful computer with 64GB of RAM and still got error in calculations because of low available memory.

The average cost of generation in the base case is \$15.1 MWhr⁻¹ while the most expensive marginal generation cost by any generator is \$29.7 MWhr⁻¹. Having the priority of supplying as much load as possible, the load shedding cost is considered equal to \$15,000 MWhr⁻¹ which is about 500 times more expensive than the most expensive marginal generation cost. Running the simulation using the proposed algorithm, the minimum cost is calculated at \$106,063,257. Compared with the base case, this cost is much higher. The main reason for this jump in the total cost is load shedding cost (penalty). Over ten scenarios, the expected amount of load shedding is 5,711 MWhr (0.0427% of total demand), and by considering the penalty cost for load shedding, the generation cost can be calculated as \$21,236,722 which is slightly (+5.2%) more than what is calculated with no outage, in TABLE 1. The additional \$1,050,853 cost in generation is a direct result of line outage, and preventive scheduling of more expensive generation to reduce load shedding as much as possible.

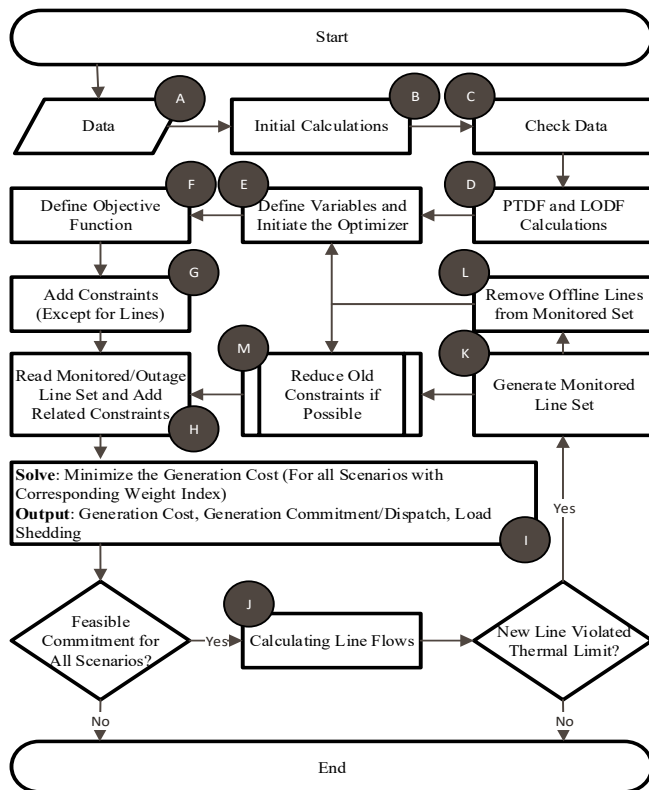


Fig. 1. Flowchart of Calculations

Using forecasted data of weather and components outage helps to minimize the load shedding and over-generation. TABLE 2 compares the load shedding and a number of critical lines (line with power flow close to its thermal limitation) for each scenario individually and the expected values for two cases. In the first case, the right-hand side of the table, the line outage possibility is not implemented, and the unit commitment problem is solved based on business as usual case. This means the system is scheduled like a typical day and later lines went out of service. The second case, the left-hand side of the table, is calculated by using the proposed algorithm

to minimize the load shedding, over-generation, and total cost. As can be seen in TABLE 2, the proposed model can reduce almost 80% of the load shedding. However, the number of critical lines in the system increased as a cost of reducing load shedding.

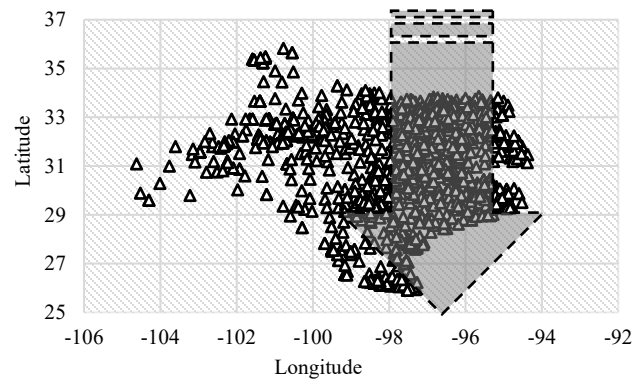


Fig. 2. Case study: Bus location (Triangles) and hurricane path (Arrow)

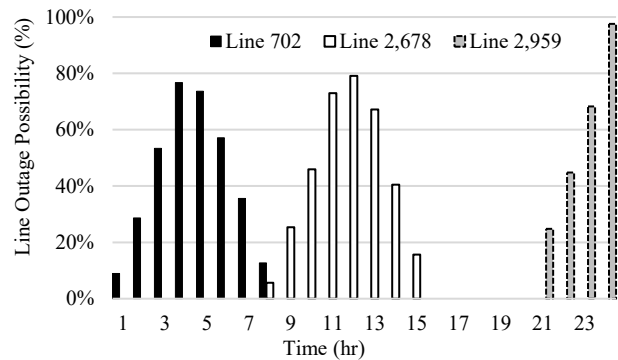


Fig. 3. Line outage possibilities for the selected lines

TABLE 1: UNIT COMMITMENT RESULTS FOR THE ORIGINAL NETWORK

Objective Function Value (\$)	Power Flow Solver	Calculation Time (Minutes)
20,185,849	BO	138
20,185,911	Shift Factor	18
20,185,869	Proposed Algorithm	2

TABLE 2: LOAD SHEDDING AND NUMBER OF CRITICAL LINES REGARDING EACH SCENARIO

	Proposed Algorithm		Business as Usual	
	Load Loss (MWhr)	Number of Critical Lines	Load Loss (MWhr)	Number of Critical Lines
Scenario 1	11,252	93	52,955	54
Scenario 2	9,028	94	45,741	52
Scenario 3	10,281	89	45,380	52
Scenario 4	8,799	88	44,170	50
Scenario 5	8,745	87	44,153	50
Scenario 6	9,001	55	39,186	15
Scenario 7	0	41	6,914	10
Scenario 8	0	40	1,113	6
Scenario 9	0	41	0	10
Scenario 10	0	41	0	10
Expected	5,711	67	27,961	31

During the simulation of many cases with different parameters, it is realized that the selected load shedding cost/penalty has a significant effect on solving the problem. As mentioned in the previous section, as the main purpose here is to supply as much demand as possible, the load shedding penalty is chosen to be a huge number. However, considering lower penalty cost for load shedding results in significantly lower required time to solve the problem and puts less lines in critical flows. TABLE 3 includes results for different values for load shedding penalty cost.

TABLE 3: RESULTS WITH DIFFERENT VALUES OF LOAD SHEDDING COSTS

Load Shedding Cost (\$/MW)	Generation Cost (\$)	Expected Load Loss (MW)	Number of Critical Lines	Calculation Time (Minutes)
15,000	21,236,722	5,711	67	423
3,000	20,854,374	6,262	62	195
1,500	20,578,476	6,430	60	80
600	20,561,629	6,743	59	72
300	20,488,406	6,945	58	31

It can be inferred from TABLE 3 that if the load shedding cost is securely higher than energy generation cost, the amount of unserved load does not change significantly, while the solution time decreases substantially and the number of critical lines would be lower.

V. CONCLUSION

Large-scale real-world power networks are among the most complex systems to operate analyze. Unit-commitment is a problem that system operators need to solve for day-ahead scheduling of the units. If the system is expected to experience exposure to a hurricane, with the possibility of component damage, stochastic unit commitment can reduce power outages. However, this problem would be hard to solve due to the large size of the system and the level of uncertainties. In this paper, an enhanced stochastic unit-commitment algorithm is proposed, which can be solved using average workstations within an acceptable time of less than 7 hours for a 2000-bus system. Obtained results for the case study show that it is possible to schedule the generation in a way that avoids 80% of the power outage by only increasing the generation cost by 5%. It should be noted that the presented analysis is only valid for transmission networks and not the distribution networks.

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